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Comments on "Heat Conduction in a Bounded, Anisotropic Medium"

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REFERENCE 1 is one of a series of very interesting papers that have been published in recent years on the development of the so-called "William's Method" (velocity or acceleration methods) for the solution of eigenvalue problems of a variety of field theories. The method is characterized by its acceptance of time-dependent boundary conditions and by its improved convergence properties relative to a conventional modal approach. Three remarks are appropriate.

1) Soon after the appearance of the Mindlin-Goodman technique² for the solution of eigenvalue problems with time-dependent boundary conditions, it became apparent³ that responses expressed according to this approach can take the form of the sum of the quasi-static solution and a product series involving eigenfunctions. Thus, as a comparison, for example, of Ref. 3 of this Comment with Ref. 2 of the Technical Note (Ref. 1) will verify, the Mindlin-Goodman technique is precisely equivalent to William's Method. There is a very substantial literature on this type of Mindlin-Goodman solution which has appeared in the last decade (e.g., Refs. 3, 4, 5, and 6). This work, which encompasses very general forms of field theories including elasticity and heat conduction, parallels in content and, in most cases, precedes the developments of William's Method. It can be concluded that those wishing to employ William's Method can usually extract the desired information from the appropriate Mindlin-Goodman type solution development.

2) Conventional modal theory for the classical field theories can accept time-dependent boundary conditions (e.g., Refs. 7, 8, 9, and 10), although convergence questions can arise.

3) These conventional modal theories can be transformed into a quasi-static solution plus the eigenfunction product series (i.e., William's solution). Vibration texts often use integration by parts¹¹ to accomplish this transformation.

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Comment on "An Automatic Node-Relabeling Scheme for Bandwidth Minimization of Stiffness Matrices"

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THE recent Note by Akyuz and Utku¹ is significant in that very little published work is available on this topic. In the case where coupling between elements is simple, the method is efficient and leads to a true minimum. However, when the coupling is more complicated, interchanges of single pairs of variables do not insure monotonic reduction of the bandwidth to a minimum. In general, the bandwidth is reduced but not necessarily to the minimum.

Suppose the current sequence of the variables is sequence A and a minimum bandwidth is given by sequence B; then all possible routes from A to B, using single interchanges, may pass through sequences that give a greater bandwidth than does sequence A. In this case the algorithm fails. For example, the plate assembly labeled in the sequence shown in Fig. 1a, and with a single variable at each node, has a stiffness matrix image of bandwidth 11 (see Fig. 1b). Using the technique of Ref. 1, the only possible single interchanges that do not increase the bandwidth in the first sweep are 3 with 4 and 9 with 10. These changes do not reduce the bandwidth and the program terminates after the next sweep.

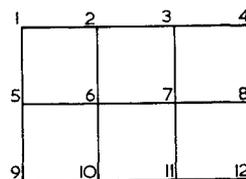


Fig. 1a Plate assembly.

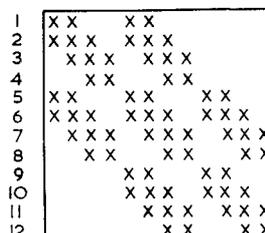


Fig. 1b Matrix image for the plate assembly.

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